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# Tracking of Multiple Maneuvering Targets using Multiscan JPDA and IMM filtering

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## Abstract

We consider the problem of tracking multiple maneuvering targets in the presence of clutter using switching multiple target motion models. A novel suboptimal filtering algorithm is developed by applying the basic interacting multiple model (IMM) approach and joint probability data association technique. But unlike the standard single scan joint probabilistic data association (JPDA) approach, we exploit a multiscan joint probabilistic data association (Mscan-JPDA) approach to solve the data association problem. The algorithm is illustrated via a simulation example involving tracking of three maneuvering targets and a multiscan data window of length two.

**Keywords:** Multitarget tracking; interacting multiple model (IMM) algorithm; multiscan joint probabilistic data association; state estimation.

## I Introduction

We consider the problem of tracking multiple maneuvering targets in presence of clutter using switching multiple target motion models. This class of problem has received considerable attention in the literature [2],[5],[6],[8],[10],[23]. The switching multiple model approach has been found to be very effective in modeling maneuvering targets [2]-[7],[9],[12]. In this approach various modes of target motion are represented by distinct kinematic models, and in a Bayesian framework, the target maneuvers are modeled by switching among these models controlled by a Markov chain. While tracking multiple targets in the presence of clutter, one has to solve the problem of measurement origin uncertainty, i.e. how to associate the data available at the sensor(s) with various targets or

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clutter (false measurements). This problem of data association has already been considered and effectively solved in the Bayesian framework by using probabilistic data association (PDA) [1],[2],[5] in the case of a single target and joint probabilistic data association (JPDA) [2],[5],[10] in the case of multiple targets. The standard JPDA algorithm uses only a single (latest) scan data available at the sensors and the state of the target is updated by using a weighted sum of measurements which could have reasonably originated from the target under consideration. To use more information to solve the data association problem, the idea of using multiple scans of data (current and past scans) seems to have been initially proposed by Drummond [19]-[21]. Drummond describes some practical issues involved but does not discuss detailed problem formulation and technical issues for multiscan JPDA. Roecker [14] has extended Drummond's ideas where he has discussed problem formulation and solution in some detail. In a simulation example presented in [14] it has been shown that performance improvement in multiple target tracking can be achieved via multiscan JPDA as compared to single scan JPDA with most of the improvement gains achieved via a window size of 2 or 3 scans.

In [14] only non-maneuvering targets (i.e. one model per target) have been considered. In this paper, we extend Roecker's approach to highly maneuvering targets where we allow multiple kinematic motion models per target. A novel suboptimal filtering algorithm is developed by applying the basic interacting multiple model (IMM) approach and multiple scan joint probability data association (Mscan-JPDA) technique. The algorithm is illustrated via a simulation example involving tracking of three maneuvering targets and multiscan data window of length two.

The paper is organized as follows. The basic multiscan JPDA problem is explained in Sec. II followed by the presentation of the problem formulation in Sec. III. The proposed IMM-based multiscan JPDA algorithm is described in Sec. IV for the case of a sliding multiscan window of size 2. A computer simulation example is presented in Sec. V.

## II Multiscan Joint Probabilistic Data Association

The first data association problem for target tracking was addressed by Bar-Shalom and Tse [1], in which they considered single target tracking in the presence of clutter. They developed the PDA, a method of data association, in which a probability weight for each measurement-to-target association is computed and the state of the target is updated with a weighted sum of measurements. For multiple target tracking in presence of clutter, the JPDA algorithm has been developed [2],[5],[10]. Unlike PDA, JPDA computes the probabilistic weight for measurement-to-target association jointly across the set of all targets and clutter. Basically it defines all the feasible joint events for the known number of targets and clutter. Each feasible joint event is a unique event that represents the association of measurements to targets and clutter. The probability of each joint

event is evaluated and the state of the target is updated with a weighted sum of joint events in which a measurement is associated with the target under consideration.

A disadvantage of JPDA is that it uses only the data present in the current scan. The idea of multiscan JPDA ( $n$ -scan-back) stems from the fact that if more scans of data are used, more information would be available for data association and hence for computing the probabilistic weights. One can use all combinations of measurements beginning from the initial time to the present time, so that all information can be utilized to compute the probabilistic weights and hence to produce the best state update. This “optimal bayesian filter” has been considered previously [2],[5]. But practically it is not feasible to implement due to memory usage and exponentially increasing computations. Hence practically feasible  $n$ -scan-back or multiscan JPDA approach will be considered with a fixed size sliding window of  $n$  scans (current and past  $n - 1$  scans).

The basic idea of multiscan JPDA can be explained as follows. In a single scan JPDA we define single scan joint events. Similarly in the multiscan scenario we define multiple-scan joint events as follows. A marginal association event  $\theta_{ir}(k)$  is said to be effective at time scan  $k$  when the validated measurement  $y_k^{(i)}$  is associated with (i.e. originates from) target  $r$  ( $r = 0, 1, \dots, N$  where  $r = 0$  means that the measurement is caused by clutter). Assuming that there are no unresolved measurements, a joint association event  $\Theta_k$  is said to be effective when a set of marginal events  $\{\theta_{ir}(k)\}$  holds true simultaneously. That is,  $\Theta_k = \bigcap_{i=1}^m \theta_{ir_i}(k)$  where  $r_i$  is the index of the target to which measurement  $y_k^{1(i)}$  is associated in the event under consideration, ( $i = 1, 2, \dots, m$ ). In the multiscan case with a scan window size  $L$  ( $L$ -scan-back) and  $k_s = k - L + s$ , we define multiscan joint events

$$\Theta_{kL} = \bigcap_{s=1}^L \bigcap_{i=1}^m \theta_{ir_{is}}(k_s)$$

where  $\theta_{ir_{is}}(k_s)$  is the marginal association event that at time scan  $k_s$ ,  $i$ th the validated measurement  $y_{k_s}^{(i)}$  is associated with target  $r_{is}$ . Let  $|\{\Theta_k\}|$  denote the total number of feasible joint events in the single scan case. In the multiscan case we get total number of multiscan feasible joint events as  $|\{\Theta_k\}| \times |\{\Theta_{k-1}\}| \times \dots \times |\{\Theta_{k-L+1}\}|$ , derived from a Cartesian product of joint events present in all scans considered in the scan window. As one can see, even for a multiscan window of length two or three, the number of feasible multiscan joint events grow exponentially. Rocker [14] and Poore [18] claim that a multiscan window of length two or three is generally enough; any further increase in the window length achieves only marginal performance improvement.

Once we define the feasible multiscan joint events, the next step is to compute the probability weights for these events. The heart of the algorithm is finding these probability weights which is discussed in detail in the sequel.

### III Problem Formulation

Assume that there are total  $N$  targets with the target set denoted as  $\mathcal{T}_N := \{1, 2, \dots, N\}$ . Assume that the dynamics of each target can be modeled as one of the  $n$  hypothesized models. The model set is denoted as  $\mathcal{M}_n := \{1, 2, \dots, n\}$ . For target  $r$  ( $r \in \mathcal{T}_N$ ), the event that model  $i$  is in effect during the sampling period  $(t_{k-1}, t_k]$  will be denoted by  $M_k^i(r)$ . Although all the targets share a common model set, any two targets may be in different motion status from time to time.

For the  $j$ -th hypothesized model (mode), the state dynamics and measurements of target  $r$  ( $r \in \mathcal{T}_N$ ) are modeled as

$$x_k(r) = F_{k-1}^j(r)x_{k-1}(r) + G_{k-1}^j(r)v_{k-1}^j(r) \quad (1)$$

and

$$z_k(r) = h^j(x_k(r)) + w_k^j(r) \quad (2)$$

where  $x_k(r)$  is the system state of target  $r$  at  $t_k$  and of dimension  $n_x$  (assuming all targets share a common state space),  $z_k(r)$  is the (true) measurement vector (i.e. due to target  $r$ ) at  $t_k$  and of dimension  $n_z$ ,  $F_{k-1}^j(r)$  and  $G_{k-1}^j(r)$  are the system matrices when model  $j$  is in effect over the sampling period  $(t_{k-1}, t_k]$  for target  $r$  and  $h^j$  is the nonlinear transformation of  $x_k(r)$  to  $z_k(r)$  for model  $j$ . A first-order linearized version of (2) is given by

$$z_k(r) = H_k^j(r)x_k(r) + w_k^j(r) \quad (3)$$

where  $H_k^j(r)$  is the Jacobian matrix of  $h^j$  evaluated at some value of the estimate of state  $x_k(r)$  (see Sec. IV). The nature of the system state, the various matrices in (1) and (3), and the measurements is specified in more detail in Sec. V. The process noise  $v_{k-1}^j(r)$  and the measurement noise  $w_k^j(r)$  are mutually uncorrelated zero-mean white Gaussian processes with covariance matrices  $Q_{k-1}^j$  (same for all targets) and  $R_k^j$  (same for all targets), respectively. At the initial time  $t_0$ , the initial conditions for the system state of target  $r$  under each model  $j$  are assumed to be Gaussian random variables with the known mean  $\bar{x}_0^j(r)$  and the known covariance  $P_0^j(r)$ . The probability of target  $r$  in model  $j$  at  $t_0$ ,  $\mu_0^j(r) = P\{M_0^j(r)\}$ , is also assumed to be known. The switching from model  $M_{k-1}^i(r)$  to model  $M_k^j(r)$  is governed by a finite-state stationary Markov chain (same for all targets) with known transition probabilities  $p_{ij} = P\{M_k^j(r)|M_{k-1}^i(r)\}$ . Henceforth,  $t_k$  will be denoted by  $k$ .

The following notations and definitions are used regarding the measurements. Note that, in general, at any time  $k$ , some measurements may be due to clutter and some due to the target, i.e. there can be more than a single measurement at time  $k$ . The measurement set (not yet validated) generated at time  $k$  is denoted as

$$Z_k := \{z_k^{(1)}, z_k^{(2)}, \dots, z_k^{(m)}\} \quad (4)$$

where  $m$  is the number of measurements generated at time  $k$ . Variable  $z_k^{(i)}$  ( $i = 1, \dots, m$ ) is the  $i$ th measurement within the set. The validated set of measurements at time  $k$  will be denoted by  $Y_k$ , containing  $\bar{m}$  ( $\leq m$ ) measurement vectors. The cumulative set of validated measurements up to time  $k$  is denoted as

$$\mathcal{Z}^k = \{Y_1, Y_2, \dots, Y_k\}. \quad (5)$$

With this problem formulation, we make the following (standard) assumptions before presenting the detailed algorithm.

**Assumptions:**

- (1) It is assumed that the number of targets ( $N$ ) is known and that for each target track has been initiated, and our objective is to maintain the tracks.
- (2) Assuming there are no unresolved measurements (i.e. measurements associated with two or more targets simultaneously), any measurement therefore is either associated with a single target or caused by clutter.
- (3) Clutter is modeled as independently and identically distributed (i.i.d.) with uniform spatial distribution over the entire validation region (across all targets).
- (4) State estimate of individual targets conditioned on the modes, joint events and set of measurements are mutually independent and Gaussian distributed i.e. states of the targets are not coupled and estimation is carried out independently.
- (5) Multiscan window of length two will be used to compute multiscan joint probabilities. Extension to higher lengths is straightforward but tedious.

The goal is to find the filtered state estimate for target  $r$  ( $r \in \mathcal{T}_N$ )

$$\hat{x}_{k|k}(r) = E\{x_k(r) | \mathcal{Z}^k\} \quad (6)$$

and the associated error covariance matrix

$$P_{k|k}(r) = E\{[x_k(r) - \hat{x}_{k|k}(r)][x_k(r) - \hat{x}_{k|k}(r)]' | \mathcal{Z}^k\} \quad (7)$$

where  $x_k(r)'$  denotes the transpose of  $x_k(r)$ .

## IV IMM/Mscan-JPDA Filtering Algorithm

We now extend the single scan IMM/JPDA filtering algorithm of [8] to apply to the multiscan case. As in [14] we will follow a sliding window multiscan approach. The approach of [8], in turn,

is based on the approaches of [2], [5], [10], [11]. As the IMM/PDAF algorithm is well-explained in [5, Sec. 4.5] and [11], the JPDAF algorithm is well-explained in [2, Sec. 9.3] and [5, Sec. 6.2], and the IMM/JPDA filter is given in detail in [8] (where all the underlying assumptions and approximations may be found in further detail), we will only briefly outline the basic steps in “one cycle” (i.e. processing needed to update for a new set of measurements and a new multiscan window) of the IMM/JPDA multiscan filter. We assume that the scan window size is two. Given state estimate at time  $k-1$  based on data up to time  $k-1$ , in Sec. IV.1 we provide first scan steps (using data up to time  $k$ ) and in Sec. IV.2 we provide the second scan steps (using data up to time  $k+1$ ).

**Assumed available:** Given the state estimate  $\hat{x}_{k-1|k-1}^j(r) := E\{x_{k-1}(r)|M_{k-1}^j(r), \mathcal{Z}^{k-1}\}$ , the associated covariance  $P_{k-1|k-1}^j(r)$  and the conditional mode probability  $\mu_{k-1}^j(r) = P[M_{k-1}^j(r)|\mathcal{Z}^{k-1}]$  at time  $k-1$  for each mode  $j \in \mathcal{M}_n$  and each target  $r \in \mathcal{T}_N$ .

#### IV.1 First Scan Steps

**Step 1.1. Interaction – mixing of the estimate from the previous time** ( $\forall j \in \mathcal{M}_n, \forall r \in \mathcal{T}_N$ ):

$$\text{predicted mode probability: } \mu_k^{j-}(r) := P\{M_k^j(r)|\mathcal{Z}^{k-1}\} = \sum_{i=1}^n p_{ij} \mu_{k-1}^i(r). \quad (8)$$

$$\text{mixing probability: } \mu^{ij}(r) := P\{M_{k-1}^i(r)|M_k^j(r), \mathcal{Z}^{k-1}\} = p_{ij} \mu_{k-1}^i(r) / \mu_k^{j-}(r). \quad (9)$$

$$\text{mixed estimate: } \hat{x}_{k-1|k-1}^{0j}(r) := E\{x_{k-1}(r)|M_k^j(r), \mathcal{Z}^{k-1}\} = \sum_{i=1}^n \hat{x}_{k-1|k-1}^i(r) \mu^{ij}(r). \quad (10)$$

covariance of the mixed estimate:

$$\begin{aligned} P_{k-1|k-1}^{0j}(r) &:= E\{[x_{k-1}(r) - \hat{x}_{k-1|k-1}^{0j}(r)][x_{k-1}(r) - \hat{x}_{k-1|k-1}^{0j}(r)]' | M_k^j(r), \mathcal{Z}^{k-1}\} \\ &= \sum_{i=1}^n \{P_{k-1|k-1}^i(r) + [\hat{x}_{k-1|k-1}^i(r) - \hat{x}_{k-1|k-1}^{0j}(r)][\hat{x}_{k-1|k-1}^i(r) - \hat{x}_{k-1|k-1}^{0j}(r)]' \} \mu^{ij}(r). \end{aligned} \quad (11)$$

**Step 1.2. Predicted state** ( $\forall j \in \mathcal{M}_n, \forall r \in \mathcal{T}_N$ ):

$$\text{State prediction: } \hat{x}_{k|k-1}^j(r) := E\{x_k(r)|M_k^j(r), \mathcal{Z}^{k-1}\} = F_{k-1}^j \hat{x}_{k-1|k-1}^{0j}(r). \quad (12)$$

State prediction error covariance:

$$\begin{aligned} P_{k|k-1}^j(r) &= E\{[x_k(r) - \hat{x}_{k|k-1}^j(r)][x_k(r) - \hat{x}_{k|k-1}^j(r)]' | M_k^j(r), \mathcal{Z}^{k-1}\} \\ &= F_{k-1}^j P_{k-1|k-1}^{0j}(r) F_{k-1}^{j'} + G_{k-1}^j Q_{k-1}^j G_{k-1}^{j'}. \end{aligned} \quad (13)$$

Using (2) and (12), the mode-conditioned predicted measurement of target  $r$  is

$$\hat{z}_k^j(r) := h^j(\hat{x}_{k|k-1}^j(r)). \quad (14)$$

Using the linearized version (3), the covariance of the mode-conditioned residual  $\nu_k^{j(i)}(r) := z_k^{(i)} - \hat{z}_k^j(r)$  is given by

$$S_k^j(r) := E\{\nu_k^{j(i)}(r)\nu_k^{j(i)}(r)' | M_k^j(r), \mathcal{Z}^{k-1}\} = H_k^j(r)P_{k|k-1}^j(r)H_k^{j'}(r) + R_k^j \quad (15)$$

where  $H_k^j(r)$  is the first order derivative (Jacobian matrix) of  $h^j(\cdot)$  at  $\hat{x}_{k|k-1}^{j(0)}(r)$ . Note that (15) assumes that  $z_k^{(i)}$  originates from the target  $r$ .

**Step 1.3. Measurement validation:** There are two steps to measurement validation. First perform measurement validation for each target  $r$  ( $r \in \mathcal{T}_N$ ) separately. For target  $r$ , the validation region is taken to be the same for all models, i.e., as the largest of them. Let ( $|A| = \det(A)$ )

$$j_r := \arg \left\{ \max_{j \in \mathcal{M}_n} |S_k^j(r)| \right\}. \quad (16)$$

Then measurement  $z_k^{(i)}$  ( $i = 1, 2, \dots, m(k)$ ) is validated if and only if

$$[z_k^{(i)} - \hat{z}_k^{j_r}(r)]' [S_k^{j_r}(r)]^{-1} [z_k^{(i)} - \hat{z}_k^{j_r}(r)] < \gamma \quad (17)$$

where  $\gamma$  is an appropriate threshold. The volume of the validation region with the threshold  $\gamma$  is

$$V_k(r) := c_{n_z} \gamma^{n_z/2} |S_k^{j_r}(r)|^{1/2} \quad (18)$$

where  $n_z$  is the dimension of the measurement and  $c_{n_z}$  is the volume of the unit hypersphere of this dimension ( $c_1 = 2, c_2 = \pi, c_3 = 4\pi/3$ , etc.). Choice of  $\gamma$  is discussed in more detail in [5, Sec. 2.3.2]. After performing the validation for each target separately, we take the validation region for the whole target set as the union of separate validation regions of all targets. As mentioned in [2] and [5], "... this approach is adopted in order to have the PDF of each false measurement the same, i.e., uniformly distributed in the entire validation region". The volume of validation region for the whole target set is approximated by

$$V_k = \sum_{r=1}^N V_k(r). \quad (19)$$

**Step 1.4. State estimation with validated measurements** ( $\forall j \in \mathcal{M}_n, \forall r \in \mathcal{T}_N$ ): From among all the raw measurements at time  $k$ , i.e.,  $Z_k := \{z_k^{(1)}, z_k^{(2)}, \dots, z_k^{(m(k))}\}$ , define the set of validated measurement for sensor 1 at time  $k$  as

$$Y_k := \{y_k^{(1)}, y_k^{(2)}, \dots, y_k^{(\overline{m}(k))}\} \quad (20)$$

where  $\overline{m}(k)$  is the total number of validated measurement at time  $k$  and

$$y_k^{(i)} := z_k^{(l_i)} \quad (21)$$

where  $1 \leq l_1 < l_2 < \dots < l_{\overline{m}(k)} \leq m(k)$  when  $\overline{m}(k) \neq 0$ . Note that all targets share a common validated measurement set  $Y_k$ .

We now consider joint probabilistic data association across targets following [2] and [5]. Define the validation matrix

$$\Omega = [\omega_{ir}] \quad i = 1, \dots, \overline{m}(k), \quad r = 0, \dots, N \quad (22)$$

where  $\omega_{ir} = 1$  if the measurement  $i$  lies in the validation gate of target  $r$ , else it is zero. A joint association event  $\Theta_k$  is represented by the event matrix

$$\widehat{\Omega}(\Theta_k) = [\widehat{\omega}_{ir}(\Theta_k)] \quad i = 1, \dots, \overline{m}(k), \quad r = 0, \dots, N \quad (23)$$

where

$$\widehat{\omega}_{ir}(\Theta_k) = \begin{cases} 1 & \text{if } \theta_{ir}(k) \in \Theta_k \\ 0 & \text{otherwise.} \end{cases} \quad (24)$$

A feasible association event is one where a measurement can have only one source

$$\sum_{r=0}^N \widehat{\omega}_{ir}(\Theta_k) = 1 \quad \forall i, \quad (25)$$

and where at most one measurement can originate from a target

$$\delta_r(\Theta_k) := \sum_{i=0}^{\overline{m}(k)} \widehat{\omega}_{ir}(\Theta_k) \leq 1 \quad \text{for } r = 1, \dots, N. \quad (26)$$

The above joint events  $\Theta_k$  are mutually exclusive and exhaustive.

Following the definitions in [2] and [5], define the binary measurement association indicator

$$\tau_i(\Theta_k) := \sum_{r=1}^N \widehat{\omega}_{ir}(\Theta_k), \quad i = 1, \dots, \overline{m}(k), \quad (27)$$

to indicate whether the validated measurement  $y_k^{(i)}$  is associated with a target in event  $\Theta_k$ . Furthermore, the number of false (unassociated) measurements in event  $\Theta_k$  is

$$\phi(\Theta_k) = \sum_{i=1}^{\overline{m}(k)} [1 - \tau_i(\Theta_k)]. \quad (28)$$

We will limit our discussion to nonparametric JPDA [2],[5]. One can evaluate the likelihood that the target  $r$  is in model  $j_r$  as

$$\begin{aligned} \Lambda_k^{j_r}(r) &:= p[Y_k | M_k^{j_r}(r), \mathcal{Z}^{k-1}] = \sum_{\Theta_k} p[Y_k | \Theta_k, M_k^{j_r}(r), \mathcal{Z}^{k-1}] P\{\Theta_k | M_k^{j_r}(r), \mathcal{Z}^{k-1}\} \\ &= \sum_{\Theta_k} p[Y_k | \Theta_k, M_k^{j_r}(r), \mathcal{Z}^{k-1}] P\{\Theta_k\} \end{aligned} \quad (29)$$



where, as in [2,(9-31)], the irrelevant conditioning terms have been omitted in the last line of (29) and the conditioning on  $\bar{m}(k)$  is implicit in the event  $\Theta_k$ . The first term in the last line of (29) can be written as

$$p[Y_k|\Theta_k, M_k^{j_r}(r), \mathcal{Z}^{k-1}] = \sum_{j_1=1}^n \cdots \sum_{j_{r-1}=1}^n \sum_{j_{r+1}=1}^n \cdots \sum_{j_N=1}^n$$

$$p[Y_k|\Theta_k, M_k^{j_1}(1), \dots, M_k^{j_{r-1}}(r-1), M_k^{j_r}(r), M_k^{j_{r+1}}(r+1), \dots, M_k^{j_N}(N), \mathcal{Z}^{k-1}]$$

$$\times P\{M_k^{j_1}(1), \dots, M_k^{j_{r-1}}(r-1), M_k^{j_{r+1}}(r+1), \dots, M_k^{j_N}(N)|\Theta_k, M_k^{j_r}(r), \mathcal{Z}^{k-1}\}. \quad (30)$$

The second term (apriori joint association probabilities) in the last line of (29) turns out to be ([5, Sec. 6.2], 2, Sec. 9.3])

$$P\{\Theta_k\} = \frac{\phi(\Theta_k)! \epsilon}{\bar{m}(k)!} \prod_{s=1}^N (P_D)^{\delta_s(\Theta_k)} (1 - P_D)^{1-\delta_s(\Theta_k)} \quad (31)$$

where  $P_D$  is the detection probability (assumed to be the same for all targets) and  $\epsilon > 0$  is a “diffuse” prior (for nonparametric modeling of clutter) whose exact value is irrelevant. We assume that the states of the targets (including the modes) conditioned on the past observations are mutually independent. Then the first term on the right-side of (30) can be written as

$$p[Y_k|\Theta_k, M_k^{j_1}(1), \dots, M_k^{j_{r-1}}(r-1), M_k^{j_r}(r), M_k^{j_{r+1}}(r+1), \dots, M_k^{j_N}(N), \mathcal{Z}^{k-1}]$$

$$\approx \prod_{i=1}^{\bar{m}(k)} p[y_k^{(i)}|\theta_{ir_i}(k), M_k^{j_{r_i}}(r_i), \mathcal{Z}^{k-1}], \quad \theta_{ir_i}(k) \subset \Theta_k, \quad (32)$$

where the conditional PDF (probability density function) of the validated measurement  $y_k^{(i)}$  given its origin and target mode, is given by

$$p[y_k^{(i)}|\theta_{ir_i}(k), M_k^{j_{r_i}}(r_i), \mathcal{Z}^{k-1}] = \begin{cases} \mathcal{N}(y_k^{(i)}; \hat{z}_k^{j_{r_i}}(r_i), S_k^{j_{r_i}}(r_i)) & \text{if } \tau_i(\Theta_k) = 1, \\ 1/V_k & \text{if } \tau_i(\Theta_k) = 0 \end{cases} \quad (33)$$

where

$$\mathcal{N}(x; y, P) := |2\pi P|^{-1/2} \exp[-\frac{1}{2}(x-y)'P^{-1}(x-y)]. \quad (34)$$

The second term on the right-side of (30) is given by

$$P\{M_k^{j_1}(1), \dots, M_k^{j_{r-1}}(r-1), M_k^{j_{r+1}}(r+1), \dots, M_k^{j_N}(N)|\Theta_k, M_k^{j_r}(r), \mathcal{Z}^{k-1}\}$$

$$= \prod_{s=1, s \neq r}^N P\{M_k^{j_s}(s)|\Theta_k, M_k^{j_r}(r), \mathcal{Z}^{k-1}\} = \prod_{s=1, s \neq r}^N P\{M_k^{j_s}(s)|\mathcal{Z}^{k-1}\} = \prod_{s=1, s \neq r}^N \mu_k^{j_s-}(s). \quad (35)$$

The probability of the joint association event  $\Theta_k$  given that model  $j$  is effective for target  $r$  from time  $k-1$  through  $k$  is

$$\begin{aligned} P\{\Theta_k | M_k^j(r), \mathcal{Z}^{k-1}, Y_k\} &= \frac{1}{c} p[Y_k | \Theta_k, M_k^j(r), \mathcal{Z}^{k-1}] P\{\Theta_k | M_k^j(r), \mathcal{Z}^{k-1}\} \\ &= \frac{1}{c} p[Y_k | \Theta_k, M_k^j(r), \mathcal{Z}^{k-1}] P\{\Theta_k\} =: \beta_k^j(r, \Theta_k) \end{aligned} \quad (36)$$

where the first term can be calculated from (30) and (32) - (35), the second term from (31), and  $c$  is a normalization constant such that  $\sum_{\Theta_k} \beta_k^j(r, \Theta_k) = 1$ .

The following updates are done for each target  $r$  ( $r \in \mathcal{T}_N$ ). Calculate  $\Lambda_k^{jr}(r)$  (needed in Step 1.5 later) via (29)-(35). Define the target and mode-conditioned innovations

$$\nu_k^j(r, \Theta_k) := \begin{cases} y_k^{(i)} - \hat{z}_k^j(r) & \text{for } i = 1, \dots, \overline{m}(k) \text{ if } \theta_{ir}(k) \subset \Theta_k \\ 0 & \text{otherwise.} \end{cases} \quad (37)$$

Using  $\hat{x}_{k|k-1}^j(r)$  (from (12)) and its covariance  $P_{k|k-1}^j(r)$  (from (13)), one computes the state update  $\hat{x}_{k|k}^j(r)$  and its covariance  $P_{k|k}^j(r)$  according to the standard PDAF [2],[5]:

$$\text{Kalman gain: } W_k^j(r) = P_{k|k-1}^j(r) H_k^{j'}(r) [S_k^j(r)]^{-1}. \quad (38)$$

$$\text{State estimate update: } \hat{x}_{k|k}^j(r) := E\{x_k(r) | M_k^j(r), \mathcal{Z}^{k-1}, Y_k\} = \sum_{\Theta_k} \hat{x}_{k|k}^j(r, \Theta_k) \beta_k^j(r, \Theta_k) \quad (39)$$

$$\hat{x}_{k|k}^j(r, \Theta_k) := E\{x_k(r) | M_k^j(r), \mathcal{Z}^{k-1}, Y_k, \Theta_k\} = \hat{x}_{k|k-1}^j(r) + W_k^j(r) \nu_k^j(r, \Theta_k). \quad (40)$$

$$\text{Covariance of } \hat{x}_{k|k}^j(r): P_{k|k}^j(r) = \sum_{\Theta_k} \sum_{i=1}^4 A_i(\Theta_k) \beta_k^j(r, \Theta_k) \quad (41)$$

where

$$A_1(\Theta_k) = E\{x_k(r) x_k'(r) | M_k^j(r), \Theta_k, \mathcal{Z}^{k-1}, Y_k\} = \hat{x}_{k|k}^j(r, \Theta_k) \hat{x}_{k|k}^{j'}(r, \Theta_k) + P_{k|k}^j(r, \Theta_k), \quad (42)$$

$$P_{k|k}^j(r, \Theta_k) = \begin{cases} P_{k|k-1}^j(r) - W_k^j(r) S_k^j(r) W_k^{j'}(r) & \text{if } \theta_{ir}(k) \subset \Theta_k \text{ for some } 1 \leq i \leq \overline{m}(k) \\ P_{k|k-1}^j(r) & \text{otherwise,} \end{cases} \quad (43)$$

$$A_2(\Theta_k) = -\hat{x}_{k|k}^j(r, \Theta_k) \hat{x}_{k|k}^{j'}(r), \quad A_3(\Theta_k) = -\hat{x}_{k|k}^j(r) \hat{x}_{k|k}^{j'}(r, \Theta_k), \quad A_4(\Theta_k) = \hat{x}_{k|k}^j(r) \hat{x}_{k|k}^{j'}(r). \quad (44)$$

**Step 1.5. Update of mode probabilities** ( $\forall j \in \mathcal{M}_n, \forall r \in \mathcal{T}_N$ ):

$$\mu_k^j(r) := P[M_k^j(r) | \mathcal{Z}^k] = P[M_k^j(r) | \mathcal{Z}^{k-1}] p[Y_k | M_k^j(r), \mathcal{Z}^{k-1}] = \frac{1}{c} \mu_k^{j-}(r) \Lambda_k^j(r) \quad (45)$$

where  $c$  is a normalization constant such that  $\sum_{j=1}^n \mu_k^j(r) = 1$ .

**Step 1.6. Combination of the mode-conditioned estimates** ( $\forall r \in \mathcal{T}_N$ ): The final state estimate update at time  $k$  is given by

$$\hat{x}_{k|k}(r) = \sum_{j=1}^n \hat{x}_{k|k}^j(r) \mu_k^j(r) \quad (46)$$

and its covariance is given by

$$P_{k|k}(r) = \sum_{j=1}^n \left\{ P_{k|k}^j(r) + [\hat{x}_{k|k}^j(r) - \hat{x}_{k|k}(r)][\hat{x}_{k|k}^j(r) - \hat{x}_{k|k}(r)]' \right\} \mu_k^j(r). \quad (47)$$

## IV.2 Second Scan Steps

Here we update to scan  $k+1$ , given data up to time  $k+1$ , with a sliding scan window of size two, scans  $\{k, k+1\}$ .

**Step 2.1. Interaction – mixing of the estimate from the previous time** ( $\forall j \in \mathcal{M}_n$ ,  $\forall r \in \mathcal{T}_N$ ):

$$\begin{aligned} \mu_k^j(r, \Theta_k) &:= P\{M_k^j(r) | \mathcal{Z}^k, \Theta_k\} = c' p[Y_k | M_k^j(r), \mathcal{Z}^{k-1}, \Theta_k] P\{M_k^j(r) | \mathcal{Z}^{k-1}, \Theta_k\} \\ &= c \beta_k^j(r, \Theta_k) \mu_k^{j-}(r). \end{aligned} \quad (48)$$

$$\text{predicted mode probability: } \mu_{k+1}^{j-}(r, \Theta_k) := P\{M_{k+1}^j(r) | \mathcal{Z}^k, \Theta_k\} = \sum_{i=1}^n p_{ij} \mu_k^i(r, \Theta_k). \quad (49)$$

$$\text{mixing probability: } \mu^{ij}(r, \Theta_k) := P\{M_k^i(r) | M_{k+1}^j(r), \mathcal{Z}^k, \Theta_k\} = p_{ij} \mu_k^i(r, \Theta_k) / \mu_{k+1}^{j-}(r, \Theta_k). \quad (50)$$

$$\text{mixed estimate: } \hat{x}_{k|k}^{0j}(r, \Theta_k) := E\{x_k(r) | M_{k+1}^j(r), \mathcal{Z}^k, \Theta_k\} = \sum_{i=1}^n \hat{x}_{k|k}^i(r, \Theta_k) \mu^{ij}(r, \Theta_k). \quad (51)$$

covariance of the mixed estimate:

$$\begin{aligned} P_{k|k}^{0j}(r, \Theta_k) &:= E\{[x_k(r) - \hat{x}_{k|k}^{0j}(r, \Theta_k)][x_k(r) - \hat{x}_{k|k}^{0j}(r, \Theta_k)]' | M_{k+1}^j(r), \mathcal{Z}^k, \Theta_k\} \\ &= \sum_{i=1}^n \{P_{k|k}^i(r, \Theta_k) + [\hat{x}_{k|k}^i(r, \Theta_k) - \hat{x}_{k|k}^{0j}(r, \Theta_k)][\hat{x}_{k|k}^i(r, \Theta_k) - \hat{x}_{k|k}^{0j}(r, \Theta_k)]' \} \mu^{ij}(r, \Theta_k). \end{aligned} \quad (52)$$

**Step 2.2. Predicted state** ( $\forall j \in \mathcal{M}_n$ ,  $\forall r \in \mathcal{T}_N$ ):

$$\text{State prediction: } \hat{x}_{k+1|k}^j(r, \Theta_k) := E\{x_{k+1}(r) | M_{k+1}^j(r), \mathcal{Z}^k, \Theta_k\} = F_k^j \hat{x}_{k|k}^{0j}(r, \Theta_k). \quad (53)$$

State prediction error covariance:

$$\begin{aligned} P_{k+1|k}^j(r, \Theta_k) &= E\{[x_{k+1}(r) - \hat{x}_{k+1|k}^j(r, \Theta_k)][x_{k+1}(r) - \hat{x}_{k+1|k}^j(r, \Theta_k)]' | M_{k+1}^j(r), \mathcal{Z}^k, \Theta_k\} \\ &= F_k^j P_{k|k}^{0j}(r, \Theta_k) F_k^{j'} + G_k^j Q_k^j G_k^{j'}. \end{aligned} \quad (54)$$

Using (2) and (53), the mode-conditioned predicted measurement of target  $r$  is

$$\hat{z}_{k+1}^j(r, \Theta_k) := h^j(\hat{x}_{k+1|k}^j(r, \Theta_k)). \quad (55)$$

Using the linearized version (3), the covariance of the mode-conditioned residual  $\nu_{k+1}^{j(i)}(r, \Theta_k) := z_{k+1}^{(i)} - \hat{z}_{k+1}^j(r, \Theta_k)$  is given by

$$\begin{aligned} S_{k+1}^j(r, \Theta_k) &:= E\{\nu_{k+1}^{j(i)}(r, \Theta_k) \nu_{k+1}^{j(i)'}(r, \Theta_k) | M_{k+1}^j(r), Z^k, \Theta_k\} \\ &= H_{k+1}^j(r, \Theta_k) P_{k+1|k}^j(r, \Theta_k) H_{k+1}^{j'}(r, \Theta_k) + R_{k+1}^j \end{aligned} \quad (56)$$

where  $H_{k+1}^j(r, \Theta_k)$  is the first order derivative (Jacobian matrix) of  $h^j(\cdot)$  at  $\hat{x}_{k+1|k}^{j(0)}(r, \Theta_k)$ .

**Step 2.3. Measurement validation:** For target  $r$ , the validation region is taken to be the same for all models and  $\Theta_k$ 's, i.e., as the largest of them. Let

$$(j_r, \bar{\Theta}_k) := \arg \left\{ \max_{j \in \mathcal{M}_n, \Theta_k} |S_{k+1}^j(r, \Theta_k)| \right\}. \quad (57)$$

Then measurement  $z_{k+1}^{(i)}$  ( $i = 1, 2, \dots, m(k+1)$ ) is validated if and only if

$$[z_{k+1}^{(i)} - \hat{z}_{k+1}^{j_r}(r, \bar{\Theta}_k)]' [S_{k+1}^{j_r}(r, \bar{\Theta}_k)]^{-1} [z_{k+1}^{(i)} - \hat{z}_{k+1}^{j_r}(r, \bar{\Theta}_k)] < \gamma \quad (58)$$

where  $\gamma$  is an appropriate threshold. The volume of the validation region with the threshold  $\gamma$  is

$$V_{k+1}(r) := c_{n_z} \gamma^{n_z/2} |S_{k+1}^{j_r}(r, \bar{\Theta}_k)|^{1/2} \quad (59)$$

where  $n_z$  is the dimension of the measurement and  $c_{n_z}$  is the volume of the unit hypersphere of this dimension. The volume of validation region for the whole target set is approximated by

$$V_{k+1} = \sum_{r=1}^N V_{k+1}(r). \quad (60)$$

**Step 2.4. State estimation with validated measurements** ( $\forall j \in \mathcal{M}_n, \forall r \in \mathcal{T}_N$ ): From among all the raw measurements at time  $k+1$ , i.e.,  $Z_{k+1} := \{z_{k+1}^{(1)}, z_{k+1}^{(2)}, \dots, z_{k+1}^{(m(k+1))}\}$ , define the set of validated measurement for sensor 1 at time  $k+1$  as

$$Y_{k+1} := \{y_{k+1}^{(1)}, y_{k+1}^{(2)}, \dots, y_{k+1}^{(\bar{m}(k+1))}\} \quad (61)$$

where  $\bar{m}(k+1)$  is the total number of validated measurement at time  $k+1$ . and

$$y_{k+1}^{(i)} := z_{k+1}^{(l_i)} \quad (62)$$

where  $1 \leq l_1 < l_2 < \dots < l_{\bar{m}(k+1)} \leq m(k+1)$  when  $\bar{m}(k+1) \neq 0$ . Note that all targets share a common validated measurement set  $Y_{k+1}$ .

We now consider joint probabilistic data association across targets as in Sec. IV.1. Define the validation matrix

$$\Omega = [\omega_{ir}] \quad i = 1, \dots, \overline{m}(k+1), \quad r = 0, \dots, N \quad (63)$$

where  $\omega_{ir} = 1$  if the measurement  $i$  lies in the validation gate of target  $r$ , else it is zero. A joint association event  $\Theta_{k+1}$  is represented by the event matrix

$$\hat{\Omega}(\Theta_{k+1}) = [\hat{\omega}_{ir}(\Theta_{k+1})] \quad i = 1, \dots, \overline{m}(k+1), \quad r = 0, \dots, N \quad (64)$$

where

$$\hat{\omega}_{ir}(\Theta_{k+1}) = \begin{cases} 1 & \text{if } \theta_{ir}(k+1) \in \Theta_{k+1} \\ 0 & \text{otherwise.} \end{cases} \quad (65)$$

A feasible association event is one where a measurement can have only one source

$$\sum_{r=0}^N \hat{\omega}_{ir}(\Theta_{k+1}) = 1 \quad \forall i, \quad (66)$$

and where at most one measurement can originate from a target

$$\delta_r(\Theta_{k+1}) := \sum_{i=0}^{\overline{m}(k+1)} \hat{\omega}_{ir}(\Theta_{k+1}) \leq 1 \quad \text{for } r = 1, \dots, N. \quad (67)$$

The above joint events  $\Theta_{k+1}$  are mutually exclusive and exhaustive.

As in Sec. IV.1, define the binary measurement association indicator

$$\tau_i(\Theta_{k+1}) := \sum_{r=1}^N \hat{\omega}_{ir}(\Theta_{k+1}), \quad i = 1, \dots, \overline{m}(k+1), \quad (68)$$

to indicate whether the validated measurement  $y_{k+1}^{(i)}$  is associated with a target in event  $\Theta_{k+1}$ . Furthermore, the number of false (unassociated) measurements in event  $\Theta_{k+1}$  is

$$\phi(\Theta_{k+1}) = \sum_{i=1}^{\overline{m}(k+1)} [1 - \tau_i(\Theta_{k+1})]. \quad (69)$$

We will limit our discussion to nonparametric JPDA [2],[5]. One can evaluate the likelihood that the target  $r$  is in model  $j_r$  as

$$\begin{aligned} \Lambda_{k+1}^{j_r}(r) &:= p[Y_{k+1} | M_{k+1}^{j_r}(r), \mathcal{Z}^k] \\ &= \sum_{\Theta_k} \sum_{\Theta_{k+1}} p[Y_{k+1} | \Theta_k, \Theta_{k+1}, M_{k+1}^{j_r}(r), \mathcal{Z}^k] P\{\Theta_k, \Theta_{k+1} | M_{k+1}^{j_r}(r), \mathcal{Z}^k\} \\ &= \sum_{\Theta_k} \sum_{\Theta_{k+1}} p[Y_{k+1} | \Theta_k, \Theta_{k+1}, M_{k+1}^{j_r}(r), \mathcal{Z}^k] P\{\Theta_{k+1}\} P\{\Theta_k | M_{k+1}^{j_r}(r), \mathcal{Z}^k\}. \end{aligned} \quad (70)$$

The first term in the last line of (70) can be written as

$$p[Y_{k+1}|\Theta_k, \Theta_{k+1}, M_{k+1}^{j_r}(r), \mathcal{Z}^k] = \sum_{j_1=1}^n \cdots \sum_{j_{r-1}=1}^n \sum_{j_{r+1}=1}^n \cdots \sum_{j_N=1}^n$$

$$p[Y_{k+1}|\Theta_k, \Theta_{k+1}, M_{k+1}^{j_1}(1), \dots, M_{k+1}^{j_{r-1}}(r-1), M_{k+1}^{j_r}(r), M_{k+1}^{j_{r+1}}(r+1), \dots, M_{k+1}^{j_N}(N), \mathcal{Z}^k]$$

$$\times P\{M_{k+1}^{j_1}(1), \dots, M_{k+1}^{j_{r-1}}(r-1), M_{k+1}^{j_{r+1}}(r+1), \dots, M_{k+1}^{j_N}(N)|\Theta_k, \Theta_{k+1}, M_{k+1}^{j_r}(r), \mathcal{Z}^k\}. \quad (71)$$

The second term (apriori joint association probabilities) in the last line of (70) turns out to be ([5, Sec. 6.2], 2, Sec. 9.3])

$$P\{\Theta_{k+1}\} = \frac{\phi(\Theta_{k+1})! \epsilon}{\bar{m}(k+1)!} \prod_{s=1}^N (P_D)^{\delta_s(\Theta_{k+1})} (1 - P_D)^{1-\delta_s(\Theta_{k+1})} \quad (72)$$

where  $P_D$  is the detection probability (assumed to be the same for all targets) and  $\epsilon > 0$  is a “diffuse” prior (for nonparametric modeling of clutter) whose exact value is irrelevant. The third term in the last line of (70) is given by

$$P\{\Theta_k|M_{k+1}^{j_r}(r), \mathcal{Z}^k\} = c' P\{M_{k+1}^{j_r}(r)|\Theta_k, \mathcal{Z}^k\} P\{\Theta_k|\mathcal{Z}^k\} = c \mu_{k+1}^{j_r-}(r, \Theta_k) P\{\Theta_k\} p[Y_k|\Theta_k, \mathcal{Z}^{k-1}] \quad (73)$$

where

$$p[Y_k|\Theta_k, \mathcal{Z}^{k-1}] = \sum_j p[Y_k|\Theta_k, M_k^j(r), \mathcal{Z}^{k-1}] \mu_k^{j-}(r). \quad (74)$$

We assume that the states of the targets (including the modes) conditioned on the past observations are mutually independent. Then the first term on the right-side of (71) can be written as

$$p[Y_{k+1}|\Theta_k, \Theta_{k+1}, M_{k+1}^{j_1}(1), \dots, M_{k+1}^{j_{r-1}}(r-1), M_{k+1}^{j_r}(r), M_{k+1}^{j_{r+1}}(r+1), \dots, M_{k+1}^{j_N}(N), \mathcal{Z}^k]$$

$$\approx \prod_{i=1}^{\bar{m}(k+1)} p[y_{k+1}^{(i)}|\theta_{ir_i}(k+1), \Theta_k, M_{k+1}^{j_{r_i}}(r_i), \mathcal{Z}^k], \quad \theta_{ir_i}(k+1) \subset \Theta_{k+1}, \quad (75)$$

where the conditional PDF of the validated measurement  $y_{k+1}^{(i)}$  given its origin and target mode, is given by

$$p[y_{k+1}^{(i)}|\theta_{ir_i}(k+1), \Theta_k, M_{k+1}^{j_{r_i}}(r_i), \mathcal{Z}^k] = \begin{cases} \mathcal{N}(y_{k+1}^{(i)}; \hat{z}_{k+1}^{j_{r_i}}(r_i, \Theta_k), S_{k+1}^{j_{r_i}}(r_i, \Theta_k)) & \text{if } \tau_i(\Theta_{k+1}) = 1, \\ 1/V_{k+1} & \text{if } \tau_i(\Theta_{k+1}) = 0. \end{cases} \quad (76)$$

The second term on the right-side of (71) is given by

$$P\{M_{k+1}^{j_1}(1), \dots, M_{k+1}^{j_{r-1}}(r-1), M_{k+1}^{j_{r+1}}(r+1), \dots, M_{k+1}^{j_N}(N)|\Theta_k, \Theta_{k+1}, M_{k+1}^{j_r}(r), \mathcal{Z}^k\}$$

$$= \prod_{s=1, s \neq r}^N P\{M_{k+1}^{j_s}(s)|\Theta_{k+1}, \Theta_k, M_{k+1}^{j_r}(r), \mathcal{Z}^k\} = \prod_{s=1, s \neq r}^N P\{M_{k+1}^{j_s}(s)|\mathcal{Z}^k, \Theta_k\}$$

$$= \prod_{s=1, s \neq r}^N \mu_{k+1}^{js-}(s, \Theta_k). \quad (77)$$

The probability of the joint association events  $\Theta_{k+1}$  and  $\Theta_k$  given that model  $j$  is effective for target  $r$  from time  $k$  through  $k+1$  is

$$\begin{aligned} P\{\Theta_{k+1}, \Theta_k | M_{k+1}^j(r), \mathcal{Z}^k, Y_{k+1}\} &= \frac{1}{c} p[Y_{k+1} | \Theta_{k+1}, \Theta_k, M_{k+1}^j(r), \mathcal{Z}^k] P\{\Theta_{k+1}\} P\{\Theta_k | M_{k+1}^j(r), \mathcal{Z}^k\} \\ &=: \beta_{k+1}^j(r, \Theta_{k+1}, \Theta_k) \end{aligned} \quad (78)$$

where the first term can be calculated from (71) and (75) - (77), the second term from (72), the third term from (73), and  $c$  is a normalization constant such that  $\sum_{\Theta_{k+1}} \sum_{\Theta_k} \beta_{k+1}^j(r, \Theta_{k+1}, \Theta_k) = 1$ .

The following updates are done for each target  $r$  ( $r \in \mathcal{T}_N$ ). Calculate  $\Lambda_{k+1}^{jr}(r)$  (needed in Step 2.5 later) via (70)-(77). Define the target and mode-conditioned innovations

$$\nu_{k+1}^j(r, \Theta_{k+1}, \Theta_k) := \begin{cases} y_{k+1}^{(i)} - \hat{z}_{k+1}^j(r, \Theta_k) & \text{for } i = 1, \dots, \bar{m}(k+1) \text{ if } \theta_{ir}(k+1) \subset \Theta_{k+1} \\ 0 & \text{otherwise.} \end{cases} \quad (79)$$

Using  $\hat{x}_{k+1|k}^j(r, \Theta_k)$  (from (53)) and its covariance  $P_{k+1|k}^j(r, \Theta_k)$  (from (54)), one computes the state update  $\hat{x}_{k+1|k+1}^j(r)$  and its covariance  $P_{k+1|k+1}^j(r)$  as follows.

$$\text{Kalman gain: } W_{k+1}^j(r, \Theta_k) = P_{k+1|k}^j(r, \Theta_k) H_{k+1}^{j'}(r, \Theta_k) [S_{k+1}^j(r, \Theta_k)]^{-1}. \quad (80)$$

$$\begin{aligned} \text{State estimate update: } \hat{x}_{k+1|k+1}^j(r) &:= E\{x_{k+1}(r) | M_{k+1}^j(r), \mathcal{Z}^k, Y_{k+1}\} \\ &= \sum_{\Theta_{k+1}} \sum_{\Theta_k} \hat{x}_{k+1|k+1}^j(r, \Theta_{k+1}, \Theta_k) \beta_{k+1}^j(r, \Theta_{k+1}, \Theta_k) \end{aligned} \quad (81)$$

$$\hat{x}_{k+1|k+1}^j(r, \Theta_{k+1}, \Theta_k) = \hat{x}_{k+1|k}^j(r, \Theta_k) + W_{k+1}^j(r, \Theta_k) \nu_{k+1}^j(r, \Theta_{k+1}, \Theta_k). \quad (82)$$

$$\text{Covariance of } \hat{x}_{k+1|k+1}^j(r): P_{k+1|k+1}^j(r) = \sum_{\Theta_{k+1}} \sum_{\Theta_k} \sum_{i=1}^4 A_i(\Theta_{k+1}, \Theta_k) \beta_{k+1}^j(r, \Theta_{k+1}, \Theta_k) \quad (83)$$

where

$$\begin{aligned} A_1(\Theta_{k+1}, \Theta_k) &= E\{x_{k+1}(r) x_{k+1}'(r) | M_{k+1}^j(r), \Theta_{k+1}, \Theta_k, \mathcal{Z}^k, Y_{k+1}\} \\ &= \hat{x}_{k+1|k+1}^j(r, \Theta_{k+1}, \Theta_k) + P_{k+1|k+1}^j(r, \Theta_{k+1}, \Theta_k), \end{aligned} \quad (84)$$

$$\begin{aligned} &P_{k+1|k+1}^j(r, \Theta_{k+1}, \Theta_k) \\ &= \begin{cases} P_{k+1|k}^j(r, \Theta_k) - W_{k+1}^j(r, \Theta_k) S_{k+1}^j(r, \Theta_k) W_{k+1}^{j'}(r, \Theta_k) & \text{if } \theta_{ir}(k+1) \subset \Theta_{k+1}, 1 \leq i \leq \bar{m}(k+1) \\ P_{k+1|k}^j(r, \Theta_k) & \text{otherwise,} \end{cases} \end{aligned} \quad (85)$$

$$A_2(\Theta_{k+1}, \Theta_k) = -\hat{x}_{k+1|k+1}^j(r, \Theta_{k+1}, \Theta_k) \hat{x}_{k+1|k+1}^{j'}(r), \quad (86)$$

$$A_3(\Theta_{k+1}, \Theta_k) = -\hat{x}_{k+1|k+1}^j(r) \hat{x}_{k+1|k+1}^{j'}(r, \Theta_{k+1}, \Theta_k), \quad (87)$$

$$A_4(\Theta_{k+1}, \Theta_k) = \hat{x}_{k+1|k+1}^j(r) \hat{x}_{k+1|k+1}^{j'}(r). \quad (88)$$

**Step 2.5. Update of mode probabilities** ( $\forall j \in \mathcal{M}_n, \forall r \in \mathcal{T}_N$ ):

$$\mu_{k+1}^j(r) := P[M_{k+1}^j(r) | \mathcal{Z}^{k+1}] = P[M_{k+1}^j(r) | \mathcal{Z}^k] p[Y_{k+1} | M_{k+1}^j(r), \mathcal{Z}^k] = \frac{1}{c} \mu_{k+1}^{j-}(r) \Lambda_{k+1}^j(r) \quad (89)$$

where  $c$  is a normalization constant such that  $\sum_{j=1}^n \mu_{k+1}^j(r) = 1$  and

$$\mu_{k+1}^{j-}(r) = \sum_{\Theta_k} \mu_{k+1}^j(r, \Theta_k) P\{\Theta_k\} p[Y_k | \Theta_k, \mathcal{Z}^{k-1}]. \quad (90)$$

**Step 2.6. Combination of the mode-conditioned estimates** ( $\forall r \in \mathcal{T}_N$ ): The final state estimate update at time  $k+1$  is given by

$$\hat{x}_{k+1|k+1}(r) = \sum_{j=1}^n \hat{x}_{k+1|k+1}^j(r) \mu_{k+1}^j(r) \quad (91)$$

and its covariance is given by

$$\begin{aligned} P_{k+1|k+1}(r) &= \sum_{j=1}^n \left\{ P_{k+1|k+1}^j(r) + [\hat{x}_{k+1|k+1}^j(r) - \hat{x}_{k+1|k+1}(r)][\hat{x}_{k+1|k+1}^j(r) - \hat{x}_{k+1|k+1}(r)]' \right\} \mu_{k+1}^j(r). \end{aligned} \quad (92)$$

## V Simulation Example

We now consider tracking three maneuvering targets in clutter. We carry out state estimation for each target using IMM multiscan JPDA with a scan window size of two and compare our results with single scan IMM/JPDA algorithm of [8].

**The True Trajectories:** Target 1 starts at location [10500 1740 40] in Cartesian coordinates in meters. The initial velocity is [-140 299.9 0] in m/s. Target stays at constant altitude with a constant speed of 331 m/s. Its trajectory is a straight line with constant velocity between 0 and 15 sec., a coordinated turn of -0.32 rad/s with a constant acceleration of 109 m/s<sup>2</sup> between 15 and 25 s, and a straight line with a constant velocity between 25 and 35 s. Target 2 starts at location [9800 1960 40] in Cartesian coordinates in meters. The initial velocity is [0 299 0] in m/s. The target stays at a constant altitude with a constant speed of 299 m/s. Its trajectory is a straight line with constant velocity between 0 and 15 sec., a coordinated turn of 0.32 rad/s with a constant acceleration of 94 m/s<sup>2</sup> between 15 and 25 s, and a straight line with a constant velocity between 25 and 35 s. Target 3 starts at location [9200 1740 40] in Cartesian coordinates in meters. The



initial velocity is  $[0 \ 299 \ 0]$  in m/s. The target stays at a constant altitude with a constant speed of 299 m/s. Its trajectory is a straight line with a constant velocity between 0 and 35 sec.

**The Target Motion Models:** The motion models are identical for all three targets. In each mode target dynamics are modeled in cartesian coordinates as  $x_k = Fx_{k-1} + Gv_{k-1}$  where state of the target is position, velocity and acceleration in each of the three Cartesian coordinates ( $x$ ,  $y$  and  $z$ ). Thus  $x_k$  is of dimension 9 ( $n_x=9$ ). Three models are considered in the following discussion. The system matrices  $F$  and  $G$  are defined as

$$F = \begin{bmatrix} F_b & 0 & 0 \\ 0 & F_b & 0 \\ 0 & 0 & F_b \end{bmatrix} \quad \text{and} \quad G = \begin{bmatrix} G_b & 0 & 0 \\ 0 & G_b & 0 \\ 0 & 0 & G_b \end{bmatrix}.$$

- Model 1: nearly constant velocity model with zero mean perturbation in acceleration.

$$F_b^1 = \begin{bmatrix} 1 & T & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad G_b^1 = \begin{bmatrix} \frac{T^2}{2} \\ T \\ 0 \end{bmatrix}$$

where  $T$  is the sampling period. The standard deviation of the process noise of  $M^1$  is  $5\text{m/s}^2$ .

- Model 2: Wiener process acceleration (nearly constant acceleration motion)

$$F_b^2 = \begin{bmatrix} 1 & T & \frac{T^2}{2} \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad G_b^2 = \begin{bmatrix} \frac{T^2}{2} \\ T \\ 1 \end{bmatrix}.$$

The standard deviation of the process noise of  $M^2$  is  $7.5\text{m/s}^2$ .

- Model 3: Wiener process acceleration (model with large acceleration increments, for the onset and termination of maneuvers). Here  $F_b^3 = F_b^2$  and  $G_b^3 = G_b^2$ . The standard deviation of the process noise of  $M^2$  is  $40\text{m/s}^2$ .

The initial model probabilities for three targets are identical:  $\mu_0^1 = 0.8$ ,  $\mu_0^2 = 0.1$  and  $\mu_0^3 = 0.1$ . The mode switching probability matrix for three targets is also identical and is given by

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.1 & 0.8 & 0.1 \\ 0.1 & 0.1 & 0.8 \end{bmatrix}.$$

**The Sensor:** A single sensor (radar) is used to obtain the measurements. The measurements are range, and azimuth and elevation angles for the radar. The range, azimuth and elevation transformations, respectively, are given by

$$r = (x^2 + y^2 + z^2)^{1/2}, \quad a = \tan^{-1}(y/x), \quad e = \tan^{-1}[z/(x^2 + y^2)^{1/2}].$$

The measurement noise  $w_k^j$  is assumed to be zero-mean white Gaussian with known covariance matrix  $R = \text{diag}[400\text{m}^2, 49\text{mrad}^2, 4\text{mrad}^2]$ . The sensor is assumed to be located at the origin of the coordinate system. The sampling interval was  $T = 1\text{s}$  and it was assumed that the probability of detection  $P_D = 0.997$ .

**The Clutter:** For generating false measurements in simulations, the clutter was assumed to be Poisson distributed with expected number of  $\lambda = 0.1/\text{m rad}^2$ . These statistics were used for generating the clutter in all simulations. However, a nonparametric clutter model was used for implementing all the algorithms for target tracking.

**Other Parameters:** The gates for setting up the validation regions for the sensor were based on the threshold  $\gamma = 16$ . With the measurement vector of dimension 3, this leads to a gate probability  $P_G = 0.9989$  (see p. 96 of [5]).

**Simulation Results:** The results were obtained from 30 Monte Carlo runs. Fig. 1 shows the true trajectories of the three targets and the distances among targets as a function of time. Fig. 2 shows the RMSE (root mean-square error) for the filtered position estimates for the three targets as a function of time. It is seen from Fig. 2 that the multiscan approach does provide a significant improvement over the single scan approach.

## VI Conclusions

We investigated the problem of tracking multiple maneuvering targets in the presence of clutter using switching multiple target motion models. A novel suboptimal filtering algorithm was developed by applying the basic interacting multiple model (IMM) approach and multiscan joint probability data association (JPDA) technique. Past work (see [14]) on this problem is restricted to non-maneuvering targets.

The algorithm was illustrated via a simulation example involving tracking of three maneuvering targets and a multiscan data window of length two. The simulation example shows a significant improvement in target position estimate by the proposed IMM multiscan JPDA (with a scan window size of two) compared to the results of the single scan IMM/JPDA algorithm of [8].

## VII References

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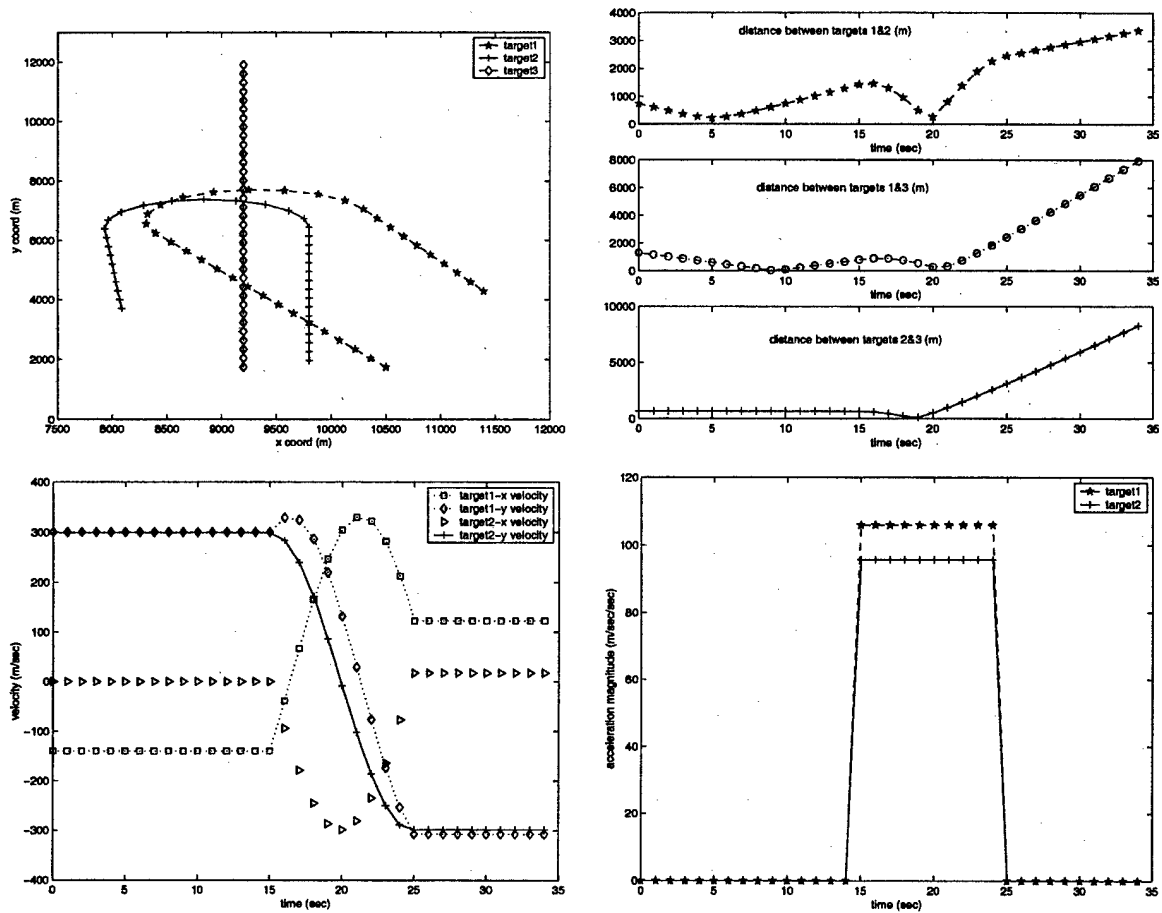


Figure 1: Trajectories of the three maneuvering targets (read left to right, top to bottom). (a) Position in  $xy$  plane. (b) distance between target pairs. (c)  $x$  and  $y$  velocities. (d) acceleration magnitudes.

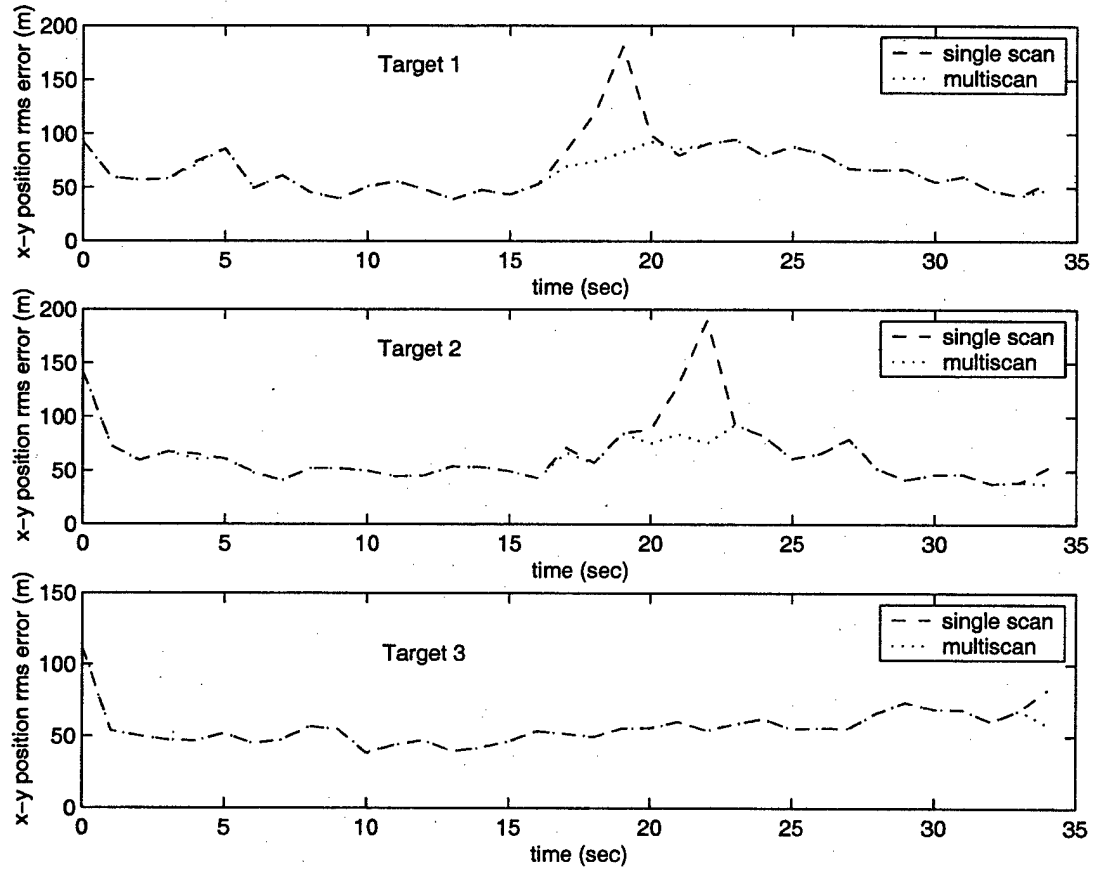


Figure 2: Root mean-square error (RMSE) in position using single scan IMM/JPDAF [8] and the proposed multiscan (window size 2 scans) IMM/Mscan-JPDAF algorithms.

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